2.1 SLOPE FIELDS

DIRECTION

SOL'N PASSES THROUGH (x_0, y_0)

GIVEN $\frac{dy}{dx} = f(x,y)$ AND $y(x_0) = y_0$.

IF THE SOL'N PASSES THROUGH (a,b)

THEN $\left.\frac{dy}{dx}\right|_{(a,b)} = f(a,b)$

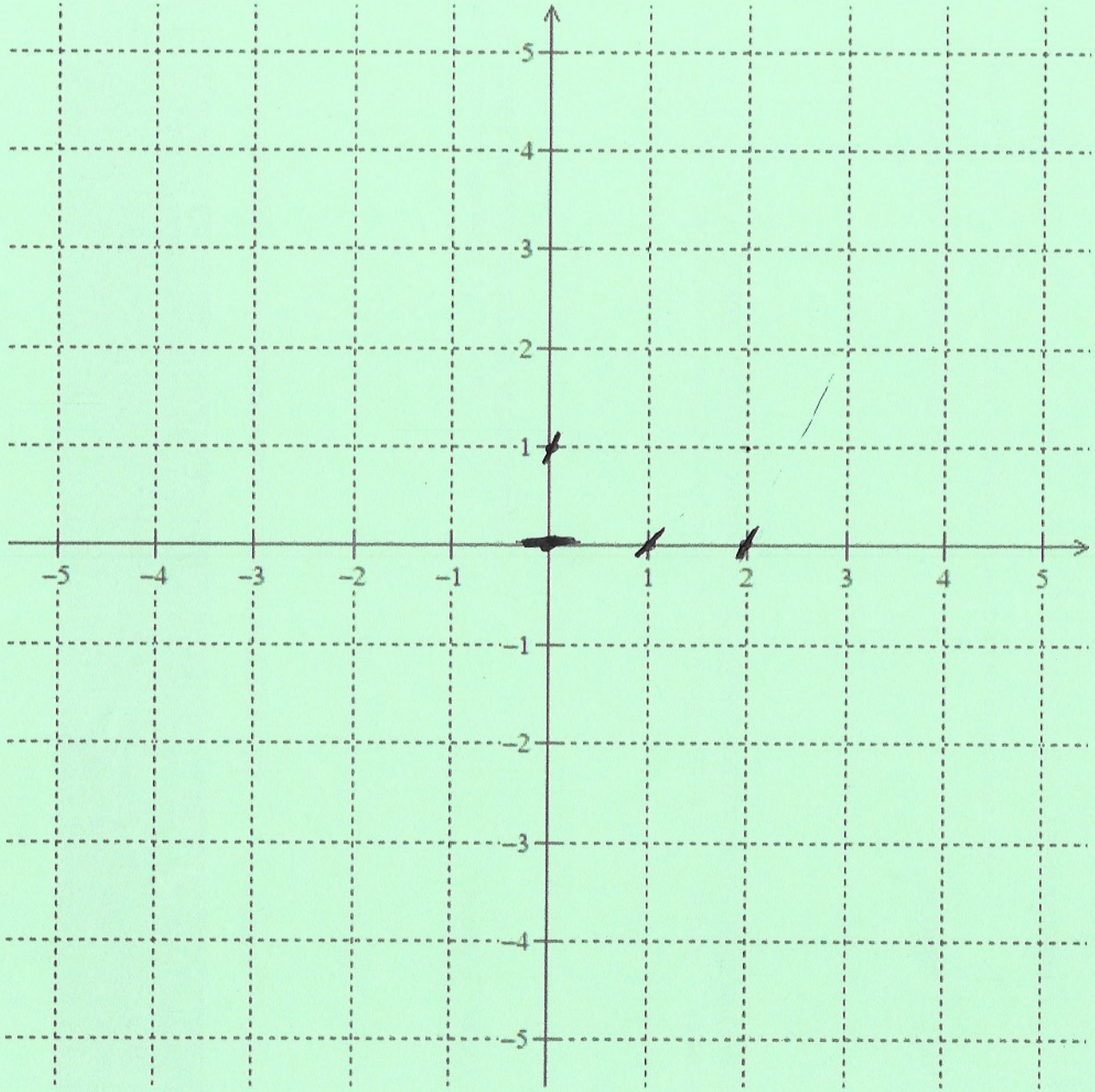
IE. THE SOL'N THROUGH (a,b)

LOOKS LIKE A LINE SEGMENT WITH SLOPE $f(a,b)$

SO TO GET A FEELING FOR SOL'NS OF DE $\frac{dy}{dx} = f(x,y)$

SKETCH THOSE LINE SEGMENTS AT MANY POINTS

LOOK LIKE



CONSIDER ODE

$$\frac{dy}{dx} = x + 2y$$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = 0 + 2 \cdot 0 = 0$$

$$\left.\frac{dy}{dx}\right|_{(1,0)} = 1 + 2 \cdot 0 = 1$$

$$\left.\frac{dy}{dx}\right|_{(2,0)} = 2 + 2 \cdot 0 = 2$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = 0 + 2 \cdot 1 = 2$$

AUTONOMOUS 1ST ORDER ODE

$$\frac{dy}{dx} = f(y)$$

eg. $\frac{dy}{dx} = y(2-y)$

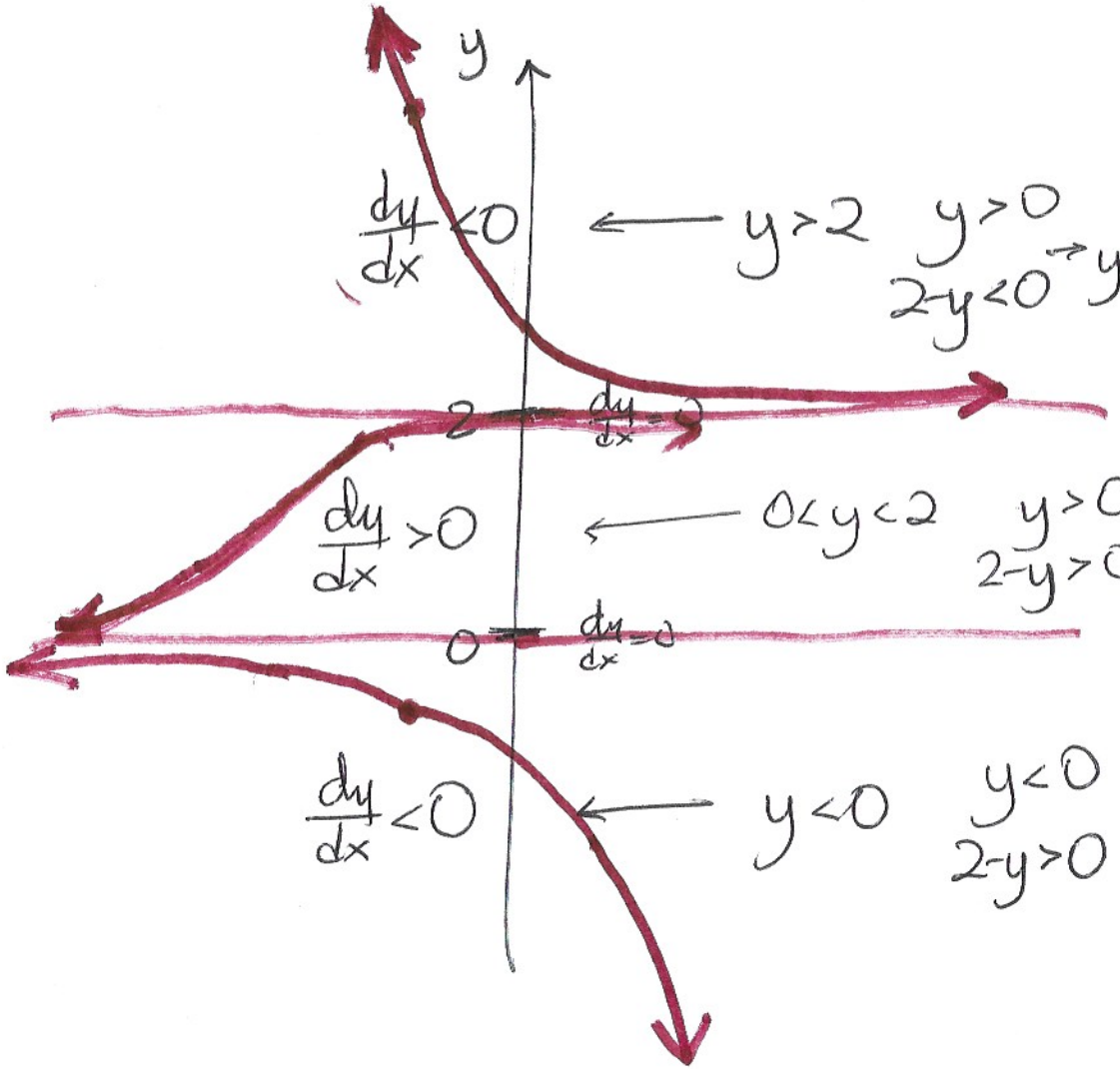
$\frac{dy}{dx} = 0$ IF $y = 0, 2$

$\downarrow$

$\frac{dy}{dx} = 0$

$y=0$ AND $y=2$ ARE EQUILIBRIUM (CONSTANT) SOLUTIONS OF $\frac{dy}{dx} = y(2-y)$

[SET $f(y)=0$ + SOLVE FOR y TO FIND EQ SOLN]



$\frac{dy}{dx} < 0 \leftarrow y > 2 \quad y > 0$

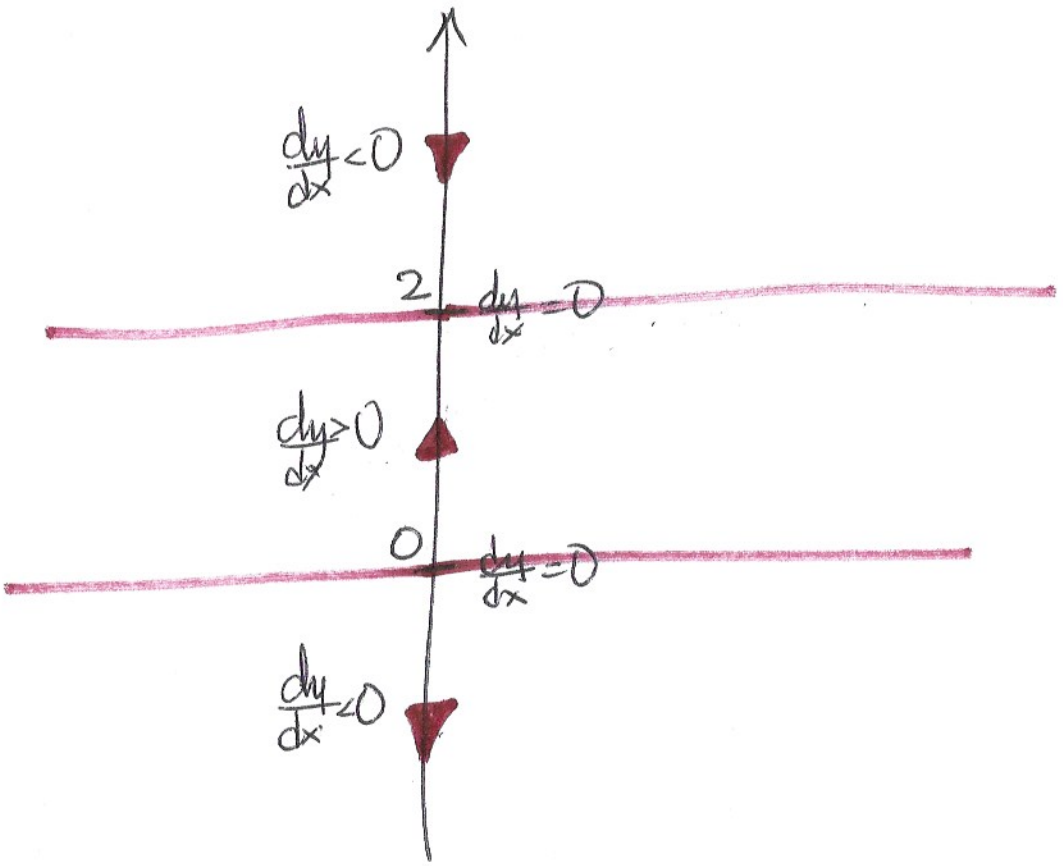
$2-y < 0 \rightarrow y(2-y) < 0$

$\frac{dy}{dx} > 0 \leftarrow 0 < y < 2 \quad y > 0$

$2-y > 0 \rightarrow y(2-y) > 0$

$\frac{dy}{dx} < 0 \leftarrow y < 0 \quad y < 0$

$2-y > 0 \rightarrow y(2-y) < 0$



IF $y(x_0)>2$

$$\lim_{x\to\infty} y(x)=2$$

IF $0<y(x_0)<2$

$$\lim_{x\to\infty} y(x)=2$$

$$\lim_{x\to-\infty} y(x)=0$$

IF $y(x_0)<0$

$$\lim_{x\to-\infty} y(x)=0$$